INCARNATE WORD ACADEMY

## AP CALCULUS

## Summer Assignment 2022-2023

To: Next Year's Calculus Students
From: TBD, Calculus Teacher

Attached is a summer homework packet, which will be due the first day of Calculus class in August.
Due to its fast pacing, there isn't always enough time to go over algebra basics in AP Calculus AB and BC. As such, the following packet is intended to help you review and hone some essential skills you have learned in Algebra and Pre-Calculus that are used frequently in Calculus. Do your future selves a favor and invest the time to master these skills now so you can focus on just the calculus later.

You will turn in the packet the first day of Calculus class, and it will count as a grade. During the first few weeks of school, you will be tested on the material in the packet.

My recommendation is that you look over the problems in the packet when you receive it but that you wait until a week or two before school starts to work the problems so that you will remember the material very well when school starts.

Remember that we will be using the TI-84 graphing calculator in Calculus. The price varies some so you should compare the prices at various places before purchasing one. Hopefully you have already purchased a TI-84, but if not, you will be expected to have a TI-84 the first week of school.

I wish you success on your Calculus journey!

NOTE: If you have the time to spare, I highly recommend spending some time getting familiar with the zoom and trace functions on your calculator. For BC, you can also look into using the parametric and polar modes of your calculator as a preview for second semester.

## Formula Sheet

Reciprocal Identities: $\quad \csc x=\frac{1}{\sin x} \quad \sec x=\frac{1}{\cos x} \quad \cot x=\frac{1}{\tan x}$
Quotient Identities: $\quad \tan x=\frac{\sin x}{\cos x} \quad \cot x=\frac{\cos x}{\sin x}$
Pythagorean Identities: $\quad \sin ^{2} x+\cos ^{2} x=1 \quad \tan ^{2} x+1=\sec ^{2} x \quad 1+\cot ^{2} x=\csc ^{2} x$

Double Angle Identities: $\quad \sin 2 x=2 \sin x \cos x$

$$
\tan 2 x=\frac{2 \tan x}{1-\tan ^{2} x}
$$

## Logarithms:

$y=\log _{a} x \quad$ is equivalent to $\quad x=a^{y}$

Product property: $\quad \log _{b} m n=\log _{b} m+\log _{b} n$

Quotient property: $\quad \log _{b} \frac{m}{n}=\log _{b} m-\log _{b} n$

$$
\text { Power property: } \quad \log _{b} m^{p}=p \log _{b} m
$$

Property of equality: If $\log _{b} m=\log _{b} n$, then $\mathrm{m}=\mathrm{n}$

Change of base formula: $\quad \log _{a} n=\frac{\log _{b} n}{\log _{b} a}$

Fractional exponent:

$$
\sqrt[b]{x^{e}}=x^{\frac{e}{b}}
$$

Negative Exponents: $\quad x^{-n}=1 / x^{n}$

$$
\begin{aligned}
\cos 2 x & =\cos ^{2} x-\sin ^{2} x \\
& =1-2 \sin ^{2} x \\
& =2 \cos ^{2} x-1
\end{aligned}
$$

The Zero Exponent: $x^{0}=1$, for x not equal to 0 .
Multiplying Powers
$\frac{\text { Multiplying Two Powers of the Same Base: }}{\left(\mathrm{X}^{\mathrm{a}}\right)\left(\mathrm{X}^{\mathrm{b}}\right)=\mathrm{X}^{(\mathrm{a}+\mathrm{b})}}$

Multiplying Powers of Different Bases:
$(x y)^{a}=\left(x^{a}\right)\left(y^{a}\right)$
Dividing Powers
Dividing Two Powers of the Same Base:
$\left(x^{a}\right) /\left(x^{b}\right)=x^{(a-b)}$
Dividing Powers of Different Bases: $(x / y)^{a}=\left(x^{a}\right) /\left(y^{a}\right)$

Slope-intercept form: $y=m x+b$
Point-slope form: $\quad y=m\left(x-x_{1}\right)+y_{1}$
Standard form: $\quad \mathrm{Ax}+\mathrm{By}+\mathrm{C}=0$

## Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

## Example:

$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}=\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1}=\frac{-7 x-7-6}{5}=\frac{-7 x-13}{5}$
$\frac{\frac{-2}{x}+\frac{3 x}{x-4}}{5-\frac{1}{x-4}}=\frac{\frac{-2}{x}+\frac{3 x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)}=\frac{-2(x-4)+3 x(x)}{5(x)(x-4)-1(x)}=\frac{-2 x+8+3 x^{2}}{5 x^{2}-20 x-x}=\frac{3 x^{2}-2 x+8}{5 x^{2}-21 x}$

Simplify each of the following.

1. $\frac{\frac{25}{a}-a}{5+a}$
2. $\frac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$
3. $\frac{4-\frac{12}{2 x-3}}{5+\frac{15}{2 x-3}}$
4. $\frac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$
5. $\frac{1-\frac{2 x}{3 x-4}}{x+\frac{32}{3 x-4}}$

## Function

To evaluate a function for a given value, simply plug the value into the function for $\mathbf{x}$.
Recall: $(f \circ g)(x)=f(g(x))$ OR $f[g(x)]$ read " $f$ of $\boldsymbol{g}$ of $\boldsymbol{x}$ " Means to plug the inside function (in this case $\mathrm{g}(\mathrm{x}))$ in for x in the outside function (in this case, $\mathrm{f}(\mathrm{x})$ ).

Example: Given $f(x)=2 x^{2}+1$ and $g(x)=x-4$ find $f(g(x))$.

$$
\begin{aligned}
f(g(x)) & =f(x-4) \\
& =2(x-4)^{2}+1 \\
& =2\left(x^{2}-8 x+16\right)+1 \\
& =2 x^{2}-16 x+32+1 \\
f(g(x)) & =2 x^{2}-16 x+33
\end{aligned}
$$

Let $f(x)=2 x+1$ and $g(x)=2 x^{2}-1$. Find each.
6. $f(2)=$ $\qquad$
7. $g(-3)=$ $\qquad$
8. $f(t+1)=$ $\qquad$
9. $f[g(-2)]=$ $\qquad$
10. $g[f(m+2)]=$ $\qquad$ 11. $\frac{f(x+h)-f(x)}{h}=$ $\qquad$

Let $f(x)=\sin x$ Find each exactly.
12. $f\left(\frac{\pi}{2}\right)=$ $\qquad$ 13. $f\left(\frac{2 \pi}{3}\right)=$

Let $f(x)=x^{2}, g(x)=2 x+5$, and $h(x)=x^{2}-1$. Find each.
14. $h[f(-2)]=$ $\qquad$
15. $f[g(x-1)]=$ $\qquad$
16. $g\left[h\left(x^{3}\right)\right]=$ $\qquad$

Find $\frac{f(x+h)-f(x)}{h}$ for the given function $\boldsymbol{f}$.
17. $f(x)=9 x+3$
18. $f(x)=5-2 x$

## Intercepts and Points of Intersection

To find the x -intercepts, let $\mathrm{y}=0$ in your equation and solve.
To find the $y$-intercepts, let $x=0$ in your equation and solve.
Example: $y=x^{2}-2 x-3$
$\frac{x-\text { int. }(\text { Let } y=0)}{0=x^{2}-2 x-3}$
$0=(x-3)(x+1)$
$x=-1$ or $x=3$
$x$ - intercepts $(-1,0)$ and $(3,0)$

$$
\begin{aligned}
& \frac{y-\text { int. }(\text { Let } x=0)}{y=0^{2}-2(0)-3} \\
& y=-3 \\
& y-\text { intercept }(0,-3)
\end{aligned}
$$

Find the x and y intercepts for each.
19. $y=2 x-5$
20. $y=x^{2}+x-2$
21. $y=x \sqrt{16-x^{2}}$
22. $y^{2}=x^{3}-4 x$

## Systems

Use substitution or elimination method to solve the system of equations.
Example:

$$
\begin{aligned}
& x^{2}+y-16 x+39=0 \\
& x^{2}-y^{2}-9=0
\end{aligned}
$$

Elimination Method
$2 x^{2}-16 x+30=0$
$x^{2}-8 x+15=0$
$(x-3)(x-5)=0$
$x=3$ and $x=5$
Plug $\mathrm{x}=3$ and $x=5$ into one original
$3^{2}-y^{2}-9=0 \quad 5^{2}-y^{2}-9=0$
$-y^{2}=0$
$y=0$
Points of Intersection $(5,4),(5,-4)$ and $(3,0)$

Substitution Method
Solve one equation for one variable.

$$
\begin{array}{ll}
y^{2}=-x^{2}+16 x-39 & (1 \text { st equation solved for } y) \\
x^{2}-\left(-x^{2}+16 x-39\right)-9=0 & \text { Plug what } y^{2} \text { is equal } \\
\text { to into second equation. } \\
2 x^{2}-16 x+30=0 & \text { (The rest is the same as } \\
x^{2}-8 x+15=0 & \text { previous example) } \\
(x-3)(x-5)=0 & \\
x=3 \text { or } x-5 &
\end{array}
$$

Find the point(s) of intersection of the graphs for the given equations.
23. $\begin{aligned} & x+y=8 \\ & 4 x-y=7\end{aligned}$
24. $\begin{aligned} & x^{2}+y=6 \\ & x+y=4\end{aligned}$
25.

$$
\begin{aligned}
& x^{2}-4 y^{2}-20 x-64 y-172=0 \\
& 16 x^{2}+4 y^{2}-320 x+64 y+1600=0
\end{aligned}
$$

## Interval Notation

26. Complete the table with the appropriate notation or graph.

| Solution | Interval Notation | Graph |
| :---: | :---: | :---: |
| $-2<x \leq 4$ |  |  |
|  | $[-1,7)$ |  |
|  |  | $\square$ |

Solve each equation. State your answer in BOTH interval notation and graphically.
27. $2 x-1 \geq 0$
28. $-4 \leq 2 x-3<4$
29. $\frac{x}{2}-\frac{x}{3}>5$

## Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.
30. $f(x)=x^{2}-5$
31. $f(x)=-\sqrt{x+3}$
32. $f(x)=3 \sin x$
33. $f(x)=\frac{2}{x-1}$

## Inverses

To find the inverse of a function, simply switch the $x$ and the $y$ and solve for the new " $y$ " value. Example:

$$
\begin{array}{ll}
f(x)=\sqrt[3]{x+1} & \text { Rewrite } \mathrm{f}(\mathrm{x}) \text { as } \mathrm{y} \\
\mathrm{y}=\sqrt[3]{x+1} & \text { Switch } \mathrm{x} \text { and } \mathrm{y} \\
\mathrm{x}=\sqrt[3]{y+1} & \text { Solve for your new } \mathrm{y} \\
(x)^{3}=(\sqrt[3]{y+1})^{3} & \text { Cube both sides } \\
x^{3}=y+1 & \text { Simplify } \\
y=x^{3}-1 & \text { Solve for } \mathrm{y} \\
f^{-1}(x)=x^{3}-1 & \text { Rewrite in inverse notation }
\end{array}
$$

Find the inverse for each function.
34. $f(x)=2 x+1$
35. $f(x)=\frac{x^{2}}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x))=g(f(x))=x$

## Example:

If: $f(x)=\frac{x-9}{4}$ and $g(x)=4 x+9$ show $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are inverses of each other.

$$
\begin{array}{rlrl}
g(f(x)) & =4\left(\frac{x-9}{4}\right)+9 & f(g(x)) & =\frac{(4 x+9)-9}{4} \\
& =x-9+9 & & =\frac{4 x+9-9}{4} \\
& =x & & =\frac{4 x}{4} \\
& & =x
\end{array}
$$

$$
f(g(x))=g(f(x))=x \text { therefore they are inverses }
$$

of each other.

Prove $f$ and $g$ are inverses of each other.
36. $f(x)=\frac{x^{3}}{2} \quad g(x)=\sqrt[3]{2 x}$
37. $f(x)=9-x^{2}, x \geq 0 \quad g(x)=\sqrt{9-x}$

## Equation of a line

Slope intercept form: $y=m x+b \quad$ Vertical line: $\mathrm{x}=\mathrm{c} \quad$ (slope is undefined)
Point-slope form: $y-y_{1}=m\left(x-x_{1}\right) \quad$ Horizontal line: $\mathrm{y}=\mathrm{c}($ slope is 0$)$
38. Use slope-intercept form to find the equation of the line having a slope of 3 and a $y$-intercept of 5 .
39. Determine the equation of a line passing through the point $(5,-3)$ with an undefined slope.
40. Determine the equation of a line passing through the point $(-4,2)$ with a slope of 0 .
41. Use point-slope form to find the equation of the line passing through the point $(0,5)$ with a slope of $2 / 3$.
42. Find the equation of a line passing through the point $(2,8)$ and parallel to the line $y=\frac{5}{6} x-1$.
43. Find the equation of a line perpendicular to the $y$ - axis passing through the point $(4,7)$.
44. Find the equation of a line passing through the points $(-3,6)$ and $(1,2)$.
45. Find the equation of a line with an $x$-intercept $(2,0)$ and a $y$-intercept $(0,3)$.

## Radian and Degree Measure

Use $\frac{180^{\circ}}{\pi \text { radians }}$ to get rid of radians and Use $\frac{\pi \text { radians }}{180^{\circ}}$ to get rid of degrees and convert to degrees. convert to radians.
46. Convert to degrees:
a. $\frac{5 \pi}{6}$
b. $\frac{4 \pi}{5}$
c. 2.63 radians
47. Convert to radians:
a. $45^{\circ}$
b. $-17^{\circ}$
c. $237^{\circ}$

## Angles in Standard Position

48. Sketch the angle in standard position.
a. $\frac{11 \pi}{6}$
b. $230^{\circ}$
c. $-\frac{5 \pi}{3}$
d. 1.8 radians

## Reference Triangles

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.
a. $\frac{2}{3} \pi$
b. $225^{\circ}$
c. $-\frac{\pi}{4}$
d. $30^{\circ}$

## Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the $y$-coordinate is the sine of the angle.

Example: $\sin 90^{\circ}=1 \quad \cos \frac{\pi}{2}=0$

50. a.) $\sin 180^{\circ}$
b.) $\cos 270^{\circ}$
c.) $\sin \left(-90^{\circ}\right)$
d.) $\sin \pi$
e.) $\cos 360^{\circ}$
f.) $\cos (-\pi)$


## Absolute Value Functions

Absolute Value functions will take whatever is inside of the absolute value and make it positive. For the following questions, use your graphing calculator to graph the given functions and state on what intervals the function is increasing, decreasing or neither. To get the absolute value symbol on your calculator go to MATH > NUM > 1:abs(
51. $y=\left|x^{2}-2\right|$
52. $y=\left|3 x^{3}+7 x^{2}-4\right|$
53. $y=\frac{|x-1|}{x-1}$
54. $y=\frac{x-1}{|x-1|}$

## Trigonometric Equations:

Solve each of the equations for $0 \leq x<2 \pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \leq x<2 \pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the beginning of the packet.)
55. $\sin x=-\frac{1}{2}$
56. $2 \cos x=\sqrt{3}$
57. $\cos 2 x=\frac{1}{\sqrt{2}}$
58. $\sin ^{2} x=\frac{1}{2}$
59. $\sin 2 x=-\frac{\sqrt{3}}{2}$
60. $2 \cos ^{2} x-1-\cos x=0$
61. $4 \cos ^{2} x-3=0$
62. $\sin ^{2} x+\cos 2 x-\cos x=0$

## Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$
\arcsin (x) \quad \sin ^{-1}(x)
$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.


## Example:

Express the value of " $y$ " in radians.
$y=\arctan \frac{-1}{\sqrt{3}} \quad$ Draw a reference triangle.


This means the reference angle is $30^{\circ}$ or $\frac{\pi}{6}$. So, $y=-\frac{\pi}{6}$ so that it falls in the interval from

$$
\frac{-\pi}{2}<y<\frac{\pi}{2} \quad \text { Answer: } \mathrm{y}=-\frac{\pi}{6}
$$

For each of the following, express the value for " $y$ " in radians.
$62.2 y=\arcsin \frac{-\sqrt{3}}{2}$
$62.4 y=\arccos (-1)$
$62.6 y=\arctan (-1)$

## Example: Find the value without a calculator.

$$
\cos \left(\arctan \frac{5}{6}\right)
$$

Draw the reference triangle in the correct quadrant first.
Find the missing side using Pythagorean Thm.
Find the ratio of the cosine of the reference triangle.

$$
\cos \theta=\frac{6}{\sqrt{61}}
$$

## For each of the following give the value without a calculator.

63. $\tan \left(\arccos \frac{2}{3}\right)$
64. $\sec \left(\sin ^{-1} \frac{12}{13}\right)$
65. $\sin \left(\arctan \frac{12}{5}\right)$
66. $\sin \left(\sin ^{-1} \frac{7}{8}\right)$

## Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x -value for which the function is undefined. That will be the vertical asymptote.
71. $f(x)=\frac{1}{x^{2}}$
72. $f(x)=\frac{x^{2}}{x^{2}-4}$
73. $f(x)=\frac{2+x}{x^{2}(1-x)}$

## Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.
Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $\mathrm{y}=0$.
Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

## Determine all Horizontal Asymptotes.

74. $f(x)=\frac{x^{2}-2 x+1}{x^{3}+x-7}$
75. $f(x)=\frac{5 x^{3}-2 x^{2}+8}{4 x-3 x^{3}+5}$
76. $f(x)=\frac{4 x^{5}}{x^{2}-7}$

## Laws of Exponents

Write each of the following expressions in the form $c a^{p} b^{q}$ where $\mathrm{c}, \mathrm{p}$ and q are constants (numbers).
75. $\frac{\left(2 a^{2}\right)^{3}}{b}$
76. $\sqrt{9 a b^{3}}$
77. $\frac{a(2 / b)}{3 / a}$
(Hint: $\sqrt{x}=x^{1 / 2}$ )
78. $\frac{a b-a}{b^{2}-b} \quad$ 79. $\frac{a^{-1}}{\left(b^{-1}\right) \sqrt{a}}$
80. $\left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^{2}\left(\frac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

## Laws of Logarithms

Simplify each of the following:
81. $\log _{2} 5+\log _{2}\left(x^{2}-1\right)-\log _{2}(x-1)$
82. $2 \log _{2} 9-\log _{2} 3$
83. $3^{2 \log _{3} 5}$
84. $\quad \log _{10}\left(10^{1 / 2}\right)$
85. $\log _{10}\left(\frac{1}{10^{x}}\right)$
86. $2 \log _{10} \sqrt{x}+\log _{10} x^{1 / 3}$

Solving Exponential and Logarithmic Equations
Solve for x. (DO NOT USE A CALCULATOR)
87. $5^{(x+1)}=25$
88. $\frac{1}{3}=3^{2 x+2}$
89. $\log _{2} x^{2}==3$
90. $\log _{3} x^{2}=2 \log _{3} 4-4 \log _{3} 5$

## Factor Completely

91. $x^{6}-16 x^{4}$
92. $4 x^{3}-8 x^{2}-25 x+50$
93. $8 x^{3}+27$
94. $x^{4}-1$

## Solve the following equations for the indicated variables:

95. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, for $a . \quad$ 96. $V=2(a b+b c+c a)$, for $a . \quad$ 97. $A=2 \pi r^{2}+2 \pi r h$, for positive $r$.

Hint: use quadratic formula
98. $A=P+x r P$, for $P$
99. $2 x-2 y d=y+x d$, for $d$
100. $\frac{2 x}{4 \pi}+\frac{1-x}{2}=0$, for $x$

Solve the equations for x :
101. $4 x^{2}+12 x+3=0$ 102. $2 x+1=\frac{5}{x+2} \quad$ 103. $\frac{x+1}{x}-\frac{x}{x+1}=0$

## Polynomial Division

104. $\left(x^{5}-4 x^{4}+x^{3}-7 x+1\right) \div(x+2)$
105. $\left(x^{5}-x^{4}+x^{3}+2 x^{2}-x+4\right) \div\left(x^{3}+1\right)$

## Summer Review Packet Answer Key

1.) $\frac{5-a}{a}$
13.) $\frac{\sqrt{3}}{2}$
2.) $\frac{2 x}{5 x+20}$
14.) 15
15.) $4 x^{2}+12 x+9$
3.) $\frac{4 x-12}{5 x}$
16.) $2 x^{6}+3$
4.) $\frac{x^{2}-x-1}{x^{2}+x+1}$
17.) 9
18.) -2
5.) $\frac{x-4}{3 x^{2}-4 x+32}$
19.) $x$-int: $\left(\frac{5}{2}, 0\right)$; $y$-int: $(0,-5)$
6.) 5
20.) $x$-int: $(-2,0)$ and ( 1,0 ); $y$-int: $(0,-2)$
7.) 17
21.) $x$-int: $(4,0)$ and $(-4,0) ; y$-int: $(0,0)$
8.) $2 t+3$
22.) $x$-int: $(2,0)$ and $(-2,0)$; $y$-int: $(0,0)$
9.) 15
10.) $8 m^{2}+40 m+49$
23.) $(3,5)$
24.) $(-1,5)$ and $(2,2)$
11.) 2
25.) $(14,-8)$ and $(6,-8)$
12.) 1
26.)

| Solution | Interval Notation | Graph |
| :---: | :---: | :---: |
|  | $(-2,4]$ |  |
| $-1 \leq x<7$ |  |  |
| $x \leq 8$ | $(-\infty, 8]$ |  |

27.) $\left[\frac{1}{2}, \infty\right)$

28.) $\left[\frac{1}{2}, \frac{7}{2}\right)$

29.) $(30, \infty)$
40.) $y=2$
30.) D: $(-\infty, \infty)$; R: $[-5, \infty)$
41.) $y=\frac{2}{3} x+5$
32.) D: $(-\infty, \infty)$; R: $[-3,3]$
33.) D: $(-\infty, 1) \cup(1, \infty)$; R: $(-\infty, 0) \cup(0, \infty)$
42.) $y=\frac{5}{6} x+\frac{19}{3}$
34.) $f^{-1}(x)=\frac{x-1}{2}$
43.) $y=7$
44.) $y=-x+3$
35.) $f^{-1}(x)= \pm \sqrt{3 x}$
36.) $f(g(x))=x ; g(f(x))=x$
37.) $f(g(x))=x ; g(f(x))=x$
38.) $y=3 x+5$
47.) a. $\frac{\pi}{4}$
b. $-\frac{17 \pi}{180}$
c. $\frac{237 \pi}{180}$
39.) $x=5$
48.) a.

b.

c.

d.

49)

50)
a) 0
b) 0
c) -1
d) 0
e) 1
f) -1

55) $x=\frac{7 \pi}{6}, \frac{11 \pi}{6}$
56) $x=\frac{\pi}{6}, \frac{11 \pi}{6}$
57) $x=\frac{\pi}{8}, \frac{7 \pi}{8}, \frac{9 \pi}{8}, \frac{15 \pi}{8}$,
58) $x=\frac{\pi}{4}, \frac{3 \pi}{4}, \frac{5 \pi}{4}, \frac{7 \pi}{4}$
59) $x=\frac{5 \pi}{6}, \frac{2 \pi}{3}, \frac{11 \pi}{6}, \frac{5 \pi}{3}$
60) $x=0, \frac{2 \pi}{3}, \frac{4 \pi}{3}$,
61) $x=\frac{\pi}{6}, \frac{5 \pi}{6}, \frac{7 \pi}{6}, \frac{11 \pi}{6}$
65) $\sin (\theta)=\frac{12}{13}$
66) $\sin (\theta)=\frac{7}{8}$
62) $x=0, \frac{\pi}{2}, \frac{3 \pi}{2}$
62.2) $x=-\frac{\pi}{3}$
62.4) $x=\pi$
62.6) $x=-\frac{\pi}{4}$
63) $\tan (\theta)=\frac{\sqrt{5}}{2}$
64) $\sec (\theta)=\frac{13}{5}$

Graph the circles and ellipses below:

71) $x=0$
72) $x=2$ and $x=-2$
73) $x=0$ and $x=1$
74) $y=0$
75) $y=-5 / 3$
76) No horizontal asymptote
75) $8 a^{6} b^{-1}$
76) $3 a^{1 / 2} b^{3 / 2}$
77) $\frac{2}{3} a^{2} b^{-1}$
78) $a b^{-1}$
79) $a^{-3 / 2} b$
80) $a^{5 / 6} b^{1 / 2}$
81) $\log _{2}(5(x+1))$
82) $3 \log _{2}(3)$
83) 25
84) $1 / 2$
85) $-x$
86) $\frac{4}{3} \log _{10}(x)$
87) $x=1$
88) $x=-3 / 2$
89) $x= \pm 2 \sqrt{2}$
90) $x= \pm \frac{4}{25}$
91) $x^{4}(x-4)(x+4)$
92) $(2 x+5)(2 x-5)(x-2)$
93) $(2 x+3)\left(4 x^{2}-6 x+9\right)$
94) $\left(x^{2}+1\right)(x+1)(x-1)$
95) $a=\frac{-x b c}{y c+z b-b c}$
96) $a=\frac{V-2 b c}{2(b+c)}$
97) $r=\frac{-\pi h+\sqrt{\pi^{2} h^{2}+2 \pi A}}{2 \pi}$
98) $P=\frac{A}{1+x r}$
99) $d=\frac{2 x-y}{2 y+x}$
100) $x=\frac{-\pi}{1-\pi}=\frac{\pi}{\pi-1}$
101) $x=\frac{-3 \pm \sqrt{6}}{2}$
102) $x=1 / 2, x=-3$
103) $x=-1 / 2$
104) $x^{4}-6 x^{3}+13 x^{2}-26 x+45-\frac{89}{x+2}$
105) $x^{2}-x+1+\frac{x^{2}+3}{x^{3}+1}$

